数学ガール

茅崎ミユ

原作：結城浩

MFコミックス
アライブシリーズ

1
Mathematical Girls
Gödel's Incompleteness
Theorems
ゲルの不完全性定
数学ガールG
茉崎ミユキ
原作 結城浩

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WE LOVE TO SPEND OUR TIME AFTER SCHOOL TIME TRAVELING THROUGH THE HISTORY OF MATHEMATICS . "CROSSING THE MATHEMATICAL PASSAGES OF TIME".
mmorn
HAVE YOU HEARD OF GÖDEL'S INCOMPLETENESS THEOREMS?

0th Period Gödel's Invitation

Gödelscher Unvollständigkeit
YUP MIRUKA. I'VE HEARD ABOUT IT.
HOWEVER, WHEN SHE'S NOT THINKING ABOUT MATHS, I CAN'T READ HER THOUGHTS AT ALL...
THIS GIRL, MIRUKA, IS MY CLASS-MATE WHO EXCELS IN MATHS.
THERE'S NO ONE THAT CAN BEAT HER IN ANYTHING MATHS-RELATED.
HM...

APPEARS
SENPAI~
MIRUKA.
AH, MIRUKA'S THINKING RIGHT NOW... I'M SORRY
THIS GIRL WHO'S LIKE A SMALL ANIMAL IS TETRA.
Ah, okay
SHE'S SIXTEEN AND ONE YEAR BELOW ME. SHE'S SHORT GIRL BUT BURSTING WITH ENERGY AND CURIOSITY.
IT'S ALRIGHT TETRA, YOU DON'T NEED TO COVER YOUR MOUTH WITH YOUR HANDS

*See note at end of chapter

GÖDEL'S INCOMPLETENESS THEOREMS COMPLETELY SHATTERED HILBERT'S PROGRAM*
RATHER, IT WAS KNOWN BY THE SHOCKING PHRASE "SHOWING THE LIMITS OF LOGIC".
フフ
LIMITS OF LOGIC... DOES THAT MEAN THERE'S A LIMIT TO MATHEMATICS !?
ガ
ガ

WELL, MANY THINGS ARE OFTEN MISUNDERSTOOD.
HOWEVER, IT IS OFTEN ESSENTIALLY REGARDED, APPROPRIATELY, AS THE "FOUNDATION OF MODERN LOGIC"
Ah senpai s--
"TO DO MATHS MATHEMATICALLY", ITS PROOFS ARE EXTREMELY CAPTIVATING
TO DO MATHS MATHEMATICALLY!?

HOW IS IT?
DOESN'T IT TICKLE YOUR CURIOSITY?
DON'T YOU GUYS WANT TO EXPLORE GÖDEL'S INCOMPLETENESS THEOREMS?

IF YOU SEE 1 1 2 3, THAT'S OUR FIBONACCI SIGN. THEN, FOLLOWING THE FIBONACCI SEQUENCE, THE NEXT SIGN IS 5
SEE YOU TOMORROW
TETRA CAME UP WITH A HAND SIGNAL FOR US MATHS LOVERS.
WE USE IT AS OUR GREET-ING.
SENPAI, LET'S DO SOME MATHS TOMOR-ROW
SURE.
SEE YOU TOMOR-ROW.
ACK
SENPAI FORGOT

I LEFT MY MECHANICAL PENCIL IN THE LIBRARY.
OH WELL.
ONII-CHAN!
HAVE YOU HEARD OF GÖDEL'S INCOMPLETENESS THEOREMS?
I only know the name though.
THIS PONYTAILED GIRL IS MY COUSIN, YURI.
SHE LIVES IN MY NEIGHBOUR-HOOD SO SHE OFTEN COMES TO HANG OUT.

STOP IT YURI
YOU'RE MY COUSIN, THERE'S NO NEED TO CLING SO TIGHTLY TO ME
THE INCOMPLETENESS THEOREMS ARE THE FUNDAMENTAL THEOREMS OF MODERN LOGIC
WHAT'S INCOM-PLETE?
THAT'S WEIRD... MATHS IS SUPPOSED TO BE PRECISE AND LOGICAL!
IN OTHER WORDS, THE MATHEMATICS THAT ONII-CHAN LOVES THE MOST HAS FLAWS?
WELL, IT'S NOT REALLY LIKE THAT..

I MUSTN'T FORGET THE CONSTRAINTS
I MUSTN'T FORGET THE CONSTRAINTS
I MUSTN'T FORGET THE CONSTRAINTS
WAH
I GOT IT WRONG AGAIN!
I THOUGHT I'D BE ABLE TO SOLVE IT USING SENPAI'S MECHANICAL PENCIL..

SENPAI!
Y-YOU FORGOT THIS YESTERDAY.
T-THEN, SEE YOU AFTER SCHOOL!
?
AH, I WAS LOOKING FOR THIS THANKS.

図書室
I THINK THAT
キーン
WE SHOULD SHOW INTRODUCTORY STUDENTS OF MATHEMATICS...
"THE TRULY ASTONISHING NUMBER OF SIMPLE AND NON-TRIVIAL THEOREMS AND RELATIONS THAT PREVAIL IN MATHEMATICS.
...IN MY OPINION, THIS PROPERTY OF MATHEMATICS SOMEHOW MIRRORS THE ORDER AND REGULARITY WHICH PREVAILS IN THE WHOLE WORLD,

AUTHOR'S NOTE TAKEN FROM "LOGICAL DILEMMAS THE LIFE AND WORK OF KURT GÖDEL" BY JOHN DAWSON JR

COME, EVERYONE.
LET'S BEGIN OUR THRILLING JOURNEY TOWARDS GÖDEL'S INCOMPLETENESS THEOREMS!

AUTHOR YUUKI HIROSHI'S EASY-TO-UNDERSTAND EXPLANATION COLUMN

0th Period

Gödel's Incompleteness Theorems

For those who have less experience with mathematics, I'll give you a quick overview of what Godel's Incompleteness Theorems and Hilbert's Program are about!

The essence of pure mathematics is built upon the assumptions that we have a set of numbers, namely the natural numbers 0, 1, 2, 3, , and that there are a set of rules of "absolute truth" (which we call axioms) which govern the processes of mathematics
By "absolute truth", I mean that it's a fact or pattern that you know works but can't really prove. For example, you know the sun will always rise from the East but you can't really prove this fact.
One example of a mathematical axiom is that
1 always follows 0,
2 always follows 1,
3 always follows 2, etc
We see and now this familiar pattern but we can't prove it. We use axioms to build or prove mathematical theories, rules or ideas
For example, by knowing the truth that the numbers 0, 1, 2, 3,... are the initial naturals and 0+0=0, we can derive the addition rule that 1+2=3.

Now Hilbert's Program claims that all knowledge in mathematics can be proved using a finite set of axioms such that none of these axioms can contradict one another. If this claim was true, then mathematicians and humanity would rejoice because it would mean that eventually man would be able to derive every truth in the universe.

However, Godel comes along in 1931 and falsifies that claim. In particular, Godel proved that there will always be a true mathematical statement that cannot be proved by established axioms. In other words, it just points out human's shortcomings, and that humans can't ever fully grasp the complete knowledge of our universe.

The gist of Gödel's argument is a quirky little paradox, the Liar paradox. Consider the statement:
"This statement is false"
Well, if the statement was true, then it would be false!
And if the statement was false, then it would be true! -
What a mind bender!!

If you want more information you can read
(Godel's Incompleteness Theorems)
http://bit.ly/godelit
and
(Hilbert's Program)
http://bit.ly/hilbertsp

Mathematical Girls
Godel s Incompleteness
Theorems

Mathematical Girls c00

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STEPPED ON THE CAT ♪
STEPPED ON THE CAT ♪
Period 1 The Kitten-Bunny Theorem
PREPARING FOR YOUR PIANO RECITAL, YUURI?
THAT'S RUDE, ONII-CHAN~

.....
THROUGH A RIGOROUS POLL
YUURI HAS BEEN SELECTED AS A GLORIOUS CAT-EARED MAID!
Hey, onii-chan!
THAT SHADY EXPRESSION ON YOUR FACE ISN'T VERY GENTLEMANLY~
WELL, LET ME REPEAT.
MY CLASS IS HOSTING A COSPLAY CAFE AT THIS WEEKEND'S CULTURAL FESTIVAL.
YUURI HAS BEEN CHOSEN TO BE A CAT-EARED MAID.
DON'T YOU THINK A LITTLE DEVIL CAFE IS INAPPROPRIATE FOR THE CULTURAL FESTIVAL?

GEEZ ONII-CHAN.
IT'S NOT LITTLE DEVIL, IT'S CAT EARS!
What's this book?
IT'S MATHS AND YET QUIZZES? HOW STRANGE!
Let's try it out!
LOGIC PUZZLES! LEND ME YOUR MECHANICAL PENCIL, ONII-CHAN.
EH? WHAT'S THIS?
AH!
IT'S ALGEBEAR!

DON'T YOU KNOW, BIG BROTHER,
IT'S THE MASCOT FOR A POPULAR MATHEMATICS EDUCATION TELEVISION PROGRAM
IT'S A LITTLE WEIRD BUT CUTE, AND IT'S REALLY POPULAR AMONG GIRLS...
HMM?
HANG ON, GIRLS...?
ANYWAY!
YOU MUST COME AND SEE YUURI'S STRIKING OUTFIT!

AND THAT'S WHY...

A COSPLAY MAID CAFE! HOW INTERESTING!
THE LIFE OF Kurt Gödel
I HAVE...
ADMIRED SENPAI'S MATHEMATICS EVER SINCE YOU GAVE A SPEECH ABOUT MATHS AT THE CULTURAL FESTIVAL.
Hmm
IN OTHER WORDS, IT'S A THEORY OF "CAT EARS => LITTLE DEVILS"
SO THAT'S WHAT MIRUKA SEES...

I'M DEBATING WHETHER I SHOULD GO TO ITS CULTURAL FESTIVAL THIS COMING SATURDAY.
OUR MIDDLE SCHOOL HELD ONE OF THOSE.
WHAT? THE CULTURAL FESTIVAL?
WHETHER CAT EARS OR LITTLE DEVILS, THEY'VE GOT NOTHING TO DO WITH OUR MAIN TOPIC OF CONVERSATION.
.....
EH!?
JUST THE TWO OF US!?
THEN, TETRA'S COMING WITH ME.
OF COURSE YOU SHOULD GO! GO!
EH
AH
...

W-WON'T YOU COME ALONG AS WELL MIRUKA?
?
WHERE ARE WE MEETING UP?
THEN LET'S SAY...

THANKS FOR WAITING TETRA.
SENPAI!
受付
ON-II CH-AN!

YUUURI
Hello
"ONII-CHAN" ...?
YUUURI IS MY COUSIN, BUT SHE'S BEEN CALLING ME THAT SINCE I WAS YOUNG.
ALGE-BEAR ...
ONII-CHAN AND THIS GIRL...
EH,
COUGH COUGH
EXCUSE ME, BUT WHAT'S YOUR RELATIONSHIP WITH ONII-CHAN?
EH? UM...
THIS GIRL IS MY JUNIOR, TETRA.

YES, I'M JUST A GIRL WHO IS HIS JUNIOR
AH, IT LOOKS LIKE MIRUKA JUST ARRIVED.
HI Miruka
ONII-CHAN IS WITH TWO GIRLS...

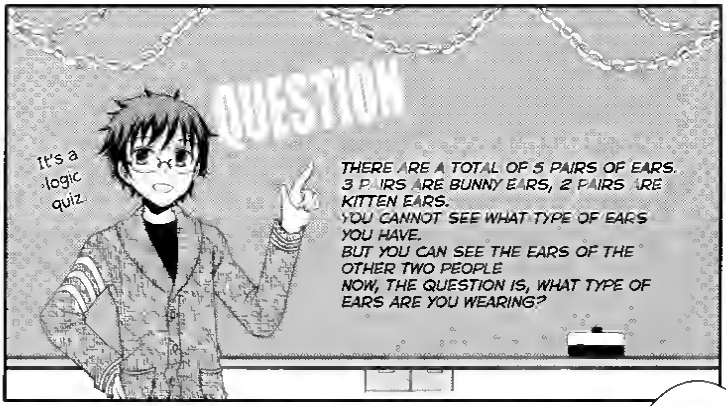
EXCUSE ME!
WHAT'S YOUR RELATIONSHIP WITH ONII-CHAN!
THIS IS THE MAIN CHARACTER FOR TODAY, MY COUSIN YUURI
HELLO YUURI
A CLASSMATE. (WHAT OF IT?)
BY THE WAY, WHY ARE YOU WEARING YOUR UNIFORM MIRUKA?
CRIES
Hm?
WE'RE VISITING A SCHOOL SO SHOULDN'T WE BE WEARING OUR UNIFORM?
Let's go
WHAT.

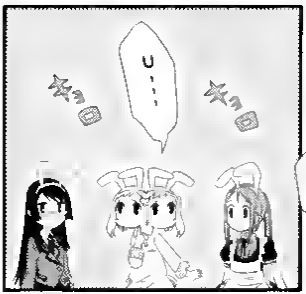
チョコバナナ
MMM
呪の教室
入口
3-3
お化け屋敷
演劇部
東京ホールA
Live!!
Rock!!
軽音楽部
ノック オン
ザ・闇の扉
入口
100円

(*TL note: mathematical union)

EH? YUURI'S EARS...
She didn't check the mirror before coming?
TA DA! THANKS FOR WAITING. ♪
WHAT'S WRONG ONII-CHAN?
THERE ARE TWO TYPES OF EARS INCLUDING THE PAIR THAT YUURI'S WEARING, THERE ARE FIVE PAIRS IN TOTAL
Bunny ears 3
Kitten ears 2
RENTAL BOX
I SEE!

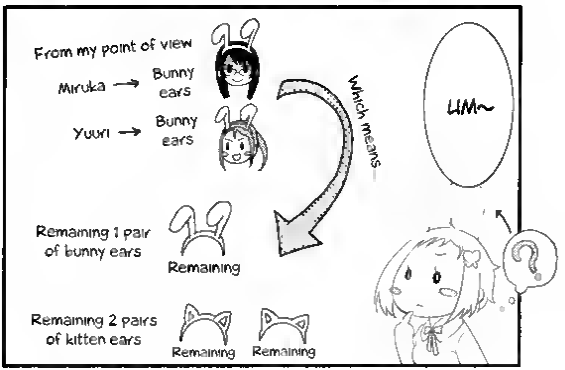
I'VE THOUGHT OF SOMETHING INTERESTING. EVERYONE, CLOSE YOUR EYES.
Sit here, Yuu!
WE'LL HIDE THE REMAINING TWO PAIRS OF EARS.
RIGHT EVERYONE, OPEN YOUR EYES.


QUESTION
It's a logic quiz.
THERE ARE A TOTAL OF 5 PAIRS OF EARS.
3 PAIRS ARE BUNNY EARS, 2 PAIRS ARE KITTEN EARS.
YOU CANNOT SEE WHAT TYPE OF EARS YOU HAVE.
BUT YOU CAN SEE THE EARS OF THE OTHER TWO PEOPLE
NOW, THE QUESTION IS, WHAT TYPE OF EARS ARE YOU WEARING?


U...

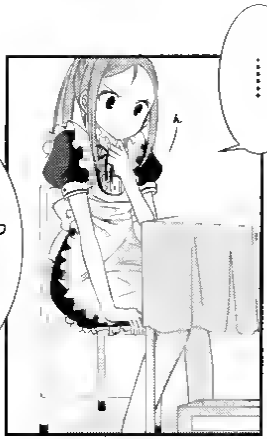
EH!
SO TETRA, UNDER THESE CONDITIONS WHAT EARS ARE YOU WEARING?

UM. I HAVE NO IDEA WHAT EARS I'M WEARING
I can't decide


From my point of view
Miruka → Bunny ears
Yuuri → Bunny ears
Which means
Remaining 1 pair of bunny ears
Remaining
Remaining 2 pairs of kitten ears
Remaining
Remaining
UM~

WHAT ABOUT MIRUKA, WHAT EARS DO YOU THINK YOU'RE WEARING?
AT THIS CURRENT POINT IN TIME, BASED ON THE INFORMATION I'VE OBTAINED, I CANNOT DRAW A CONCLUSION.
THEN, WHAT EARS ARE YOU WEARING YUURI?
AS EXPECTED OF MIRUKA ...

BOTH TETRA AND I SAW WHAT THE OTHER TWO PEOPLE ARE WEARING AND YET CANNOT GIVE AN ANSWER
THIS CASE REQUIRES SOME THOUGHT.


……

THERE WON'T BE ANY KITTEN EARS LEFT
SO I'D BE WEARING BUNNY EARS!
えっ！

THERE ARE 3 PAIRS OF BUNNY EARS AND 2 PAIRS OF KITTEN EARS
SO, IF YOU SAW THE OTHER TWO PEOPLE BOTH WEARING KITTEN EARS, THEN THAT MEANS…?
×2
×3

IN THAT CASE,
Don't know
Both wearing kitten ears
THE PEOPLE THAT SAID THEY DIDN'T KNOW MUST'VE SEEN THE OTHER TWO
NOT BOTH OF THEM WEARING KITTEN EARS...
IN OTHER WORDS, THE OTHER TWO SEEN BY THOSE WHO SAID THEY DIDN'T KNOW
AT LEAST ONE OF THEM MUST HAVE BUNNY EARS.
WOAH ...
AMAZING!
THERE'S ACTUALLY A THEOREM THAT CAN BE USED TO SOLVE THIS QUIZ
HM.

WE'LL NAME THIS THE KITTEN-BUNNY THEOREM THEN.
ねこうさねこうさねこうさねこうさねこうさねこうさねこうさねこうさねこうさねこうさ
……
SO THEN,
BASED ON THIS REASONING, WHAT EARS ARE YUURI WEARING?
UM, I DUNNO …
NYAA NYAA
WAIT A MOMENT!
DID YUURI NOTICE IT ALL BY HERSELF ?

CALM DOWN AND THINK ABOUT IT CAREFULLY, TETRA
WHEN MIRUKA ANSWERED THAT SHE DIDN'T KNOW,
WHAT KIND OF INFORMATION DID SHE GATHER?
I SEE!
I GET IT! I GET IT NOW ONII-CHAN!
FUEEEEE? W-WHAT DO YOU MEAN?
UM, SHE SAW TETRA AND YUURI'S EARS...
I SEE! WHEN I ANSWERED "I DON'T KNOW" THAT'S ANOTHER HINT!
THAT'S RIGHT.
WHEN TETRA ANSWERED "I DON'T KNOW", BY THE KITTEN-BUNNY THEOREM,
SO THEN, IF I SAW YUURI WEARING KITTEN EARS THEN I WOULD BE CERTAIN THAT I AM WEARING BUNNY EARS.
BETWEEN YUURI AND ME, AT LEAST ONE OF US IS WEARING BUNNY EARS.
HOWEVER, RIGHT NOW I AM UNABLE TO TELL WHAT TYPE OF EARS I AM WEARING.

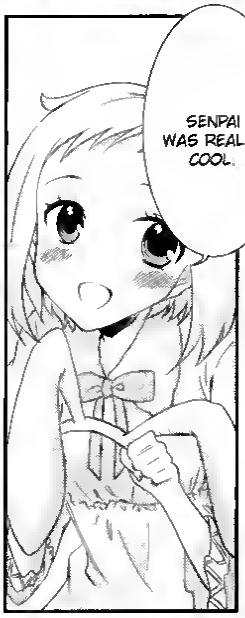
YUURI, YUURI'S EARS MUST BE BUNNY EARS!
IN OTHER WORDS
THAT'S AMAZING! YUURI! YOU ANSWERED CORRECTLY!
Yuuri wasn't wearing kitten ears

MIRUKA-SAMA?
キラキラ
BUT JUST FROM ESTABLISHING THAT "AT LEAST ONE PERSON IS WEARING BUNNY EARS" FROM THE NEGATION OF "BOTH PEOPLE ARE WEARING KITTEN EARS"
MIRUKA-SAMA SURE IS AMAZING PIECING THE INFORMATION TOGETHER!
MIRUKA-SAMA WAS ABLE TO ESTABLISH A THEORY EVEN THOUGH IT'S JUST A LOGIC QUIZ, HER MATHS SKILLS ARE WONDROUS!
NO, LOGIC IS EXTREMELY IMPORTANT IN MATHEMATICS.
GÖDEL'S INCOMPLETENESS THEOREMS ARE A FOUNDATION OF LOGIC.

IS THAT A PROBLEM?

MIRUKA SURE IS AMAZING BUT YOU'RE STILL WEARING BUNNY EARS ON YOUR HEAD.

TO BE ABLE TO EXPLAIN A DIFFICULT CONCEPT IN AN EASY TO UNDERSTAND WAY, ONII-CHAN IS ALSO SUPURRRB!


SENPAI WAS REALLY COOL.

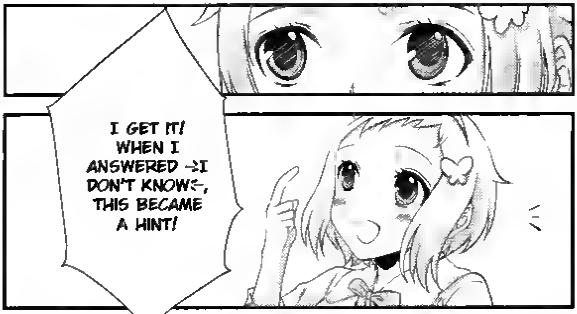
Hn

S-SINCE WHEN DID SO MANY PEOPLE COME...!?
menu
MATHS IS REALLY INTEREST-ING!
PLEASE GIVE US ANOTHER QUIZ!

TODAY'S CULTURAL FESTIVAL WAS REALLY FUN
MIDDLE SCHOOL STUDENTS ...
MATHE-MATICS ...
CULTURAL FESTIVAL...
PING
I GOT IT!
19
20
21
26
27
28
I'VE THOUGHT OF SOMETHING GOOD!

原作者・結城浩先生のよくわかる解説コラム

1ST PERIOD LOGIC QUIZ



THE CAT EARS-BUNNY EARS LOGICAL QUIZ IS INTERESTING, YET IT IS DIFFICULT.

IN MY OPINION THOSE WHO SOLVE IT AFTER FORTHRIGHTLY HEARING THE EXPLAHATION ONCE ARE FEW. I COULDN'T UNDERSTAND ANYTHING AT THE BEGINNING EITHER. AT THE TIME OF READING A LOGIC QUIZ, IT IS IMPORTANT TO REVIEW THE DETAILS OF THE STORY STEP BY STEP AS YOU PROCEED. IT IS OKAY TO WRITE ON PAPER WHEN IT GETS PUZZLING.

THE REMARK THAT MIRUKA-SAN SAID IN THE 1ST PERIOD "IT IS IMPOSSIBLE TO DETERMINE WHICH ONE IT IS" IS VERY INTERESTING. ALTHOUGH USUALLY "THINGS YOU KNOW" FORM A PATH TOWARDS RESOLUTION, IN THIS LOGIC QUIZ "I DON'T KNOW" IS WHAT BECOMES A HINT FOR THE FOLLOWING PERSON.

ALSO, IN THE MIDDLE OF THINKING ABOUT THE LOGICAL QUIZ, TETRA-CHAN BECAME AWARE THAT "THE KITTEN-BUNNY THEOREM" (IT COULD BE SAID THAT AT LEAST ONE OF THE OTHER TWO SEEN BY THOSE WHO SAID "I DON'T KNOW" HAS BUNNY EARS), THAT MIRUKA-SAN CONCLUDED, IS VERY IMPORTANT. THEY DIDN'T THINK SEPARATELY FROM SCRATCH, AND THAT'S BECAUSE THEY WERE ABLE TO HAVE THIS KITTEN-BUNNY THEOREM AS A CLUE AS THEY WERE THINKING. THOSE GIRLS, BY DISCUSSING IT TOGETHER, GET A BETTER UNDERSTANDING AS THEY CONTINUE. THERE LIES DISCOVERY, PARAPHRASING, QUESTIONS, ANSWERS AND ALSO CONCLUSIONS.

HAVING FRIENDS WHO DISCUSS MATHEMATICS ENJOYABLY YET SERIOUSLY AT THE SAME TIME IS A REALLY LOVELY THING.

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Mathematical Girls
Godel s Incompleteness
Theorems

Mathematical Girls
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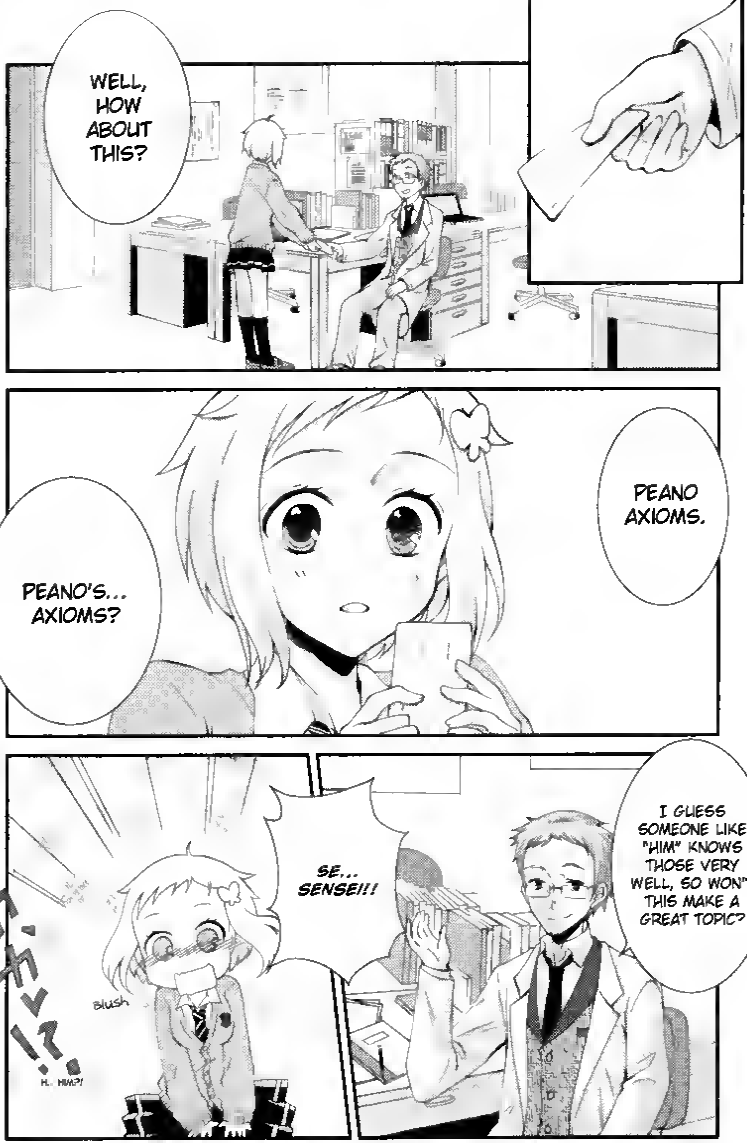
Period 2: **The game of pretending not to know**


WELL, HOW ABOUT THIS?
PEANO AXIOMS.
PEANO'S... AXIOMS?
I GUESS SOMEONE LIKE "HIM" KNOWS THOSE VERY WELL, SO WON'T THIS MAKE A GREAT TOPIC?
SE... SENSE!!!
Blush
H... HIM?!

I WILL!
THANK YOU VERY MUCH!
SO REGARDING THE LECTURER'S ROLE, DO YOUR BEST AT PERSUADING
I WILL MAKE SURE TO GET PERMISSION FOR THE "CLASSROOM" MATTER FROM EARLIER
step
step
sow

STEP
STEP
WELL, THAT'S YOUTH.
Setting questions for them is quite enjoyable
scratch, scratch
OOPS, I HAVE TO DO MY WORK TOO

SO YOU WERE HERE, SENPAI.
TETRA-CHAN.
TSUNOMIYA-SAN TOO, GOOD MORNING.
GOOD MORNING.
SMILE
Top student in his grade
grab
OH, THE RUMORED STALKER-CHAN ENTERS THE STAGE, EH?
NOW, THIS HINDRANCE HAD BETTER LEAVE QUICKLY.

SEE YOU LATER.
YOU LADY-KILLER.
PUNCH!!
Ouch Te Tetra-chan, what's the matter?
WAAH... DID I DISTURB YOU?
ow ouch
NO, REALLY, YOU DIDN'T DISTURB ME AT ALL.

SENPAI!
TO BE HONEST... WELL... THERE'S A FAVOR I WOULD LIKE TO ASK OF YOU..
?
I WENT TO MURAKI-SENSEI'S PLACE EARLIER..
W.. WELL...

MURAKI-SENSEI'S PLACE?
WHAT KIND OF CARD DID YOU GET?!
EH... UM... IT'S NOT ABOUT THE CARD...
NEVER MIND, HERE IT IS.
MURAKI-SENSEI IS OUR MATHS TEACHER.
HE IS A QUEER TEACHER WHO USUALLY WRITES PROBLEMS ON CARDS AND GIVES THEM TO US,
EVEN THOUGH THEY NOTHING TO DO WITH OUR CLASSWORK.
Peano Axioms
PA1 $1 \in \mathbb{N}$
PA2 $\forall n \in \mathbb{N}\ [n' \in \mathbb{N}]$
PA3 $\forall n \in \mathbb{N}\ [n' \neq 1]$
PA4 $\forall m \in \mathbb{N}\ \forall n \in \mathbb{N}\ [m'=n' \Rightarrow m=n]$
PA5 $(P(1) \wedge \forall k \in \mathbb{N}\ [P(k) \Rightarrow P(k')]) \Rightarrow \forall n \in \mathbb{N}\ [P(n)]$
Oh, the back has a Boolean equation version
I SEE, PEANO'S AXIOMS THIS TIME, EH...? AS EXPECTED OF MURAKI-SENSEI. HE REALLY GIVES NICE TOPICS.

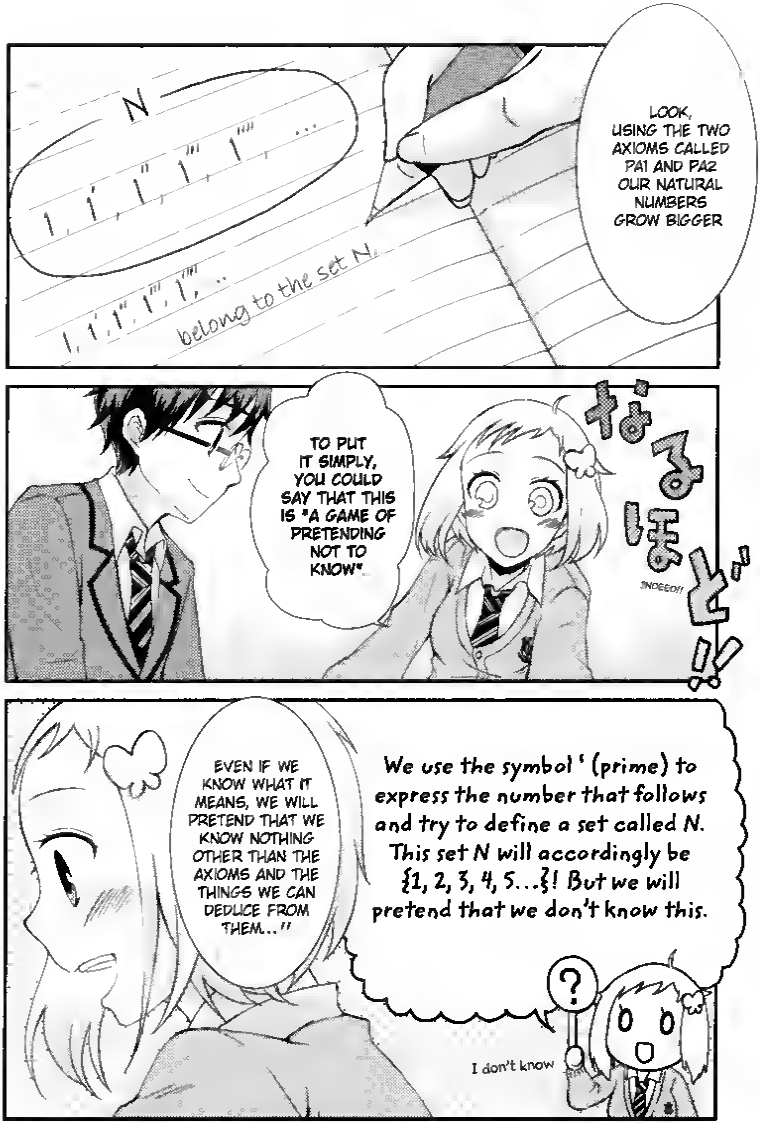
IS THAT SO?
EVEN THOUGH I READ WHAT WAS WRITTEN HERE,
I COULDN'T UNDERSTAND WHAT KIND OF THING IT WAS AT ALL. .

T/N THIS GIRL IS SPEAKING OSAKA DIALECT IN ALL HER LINES

Press
Quick
THEY DO IT EVERY DAY WITHOUT GETTING TIRED.
JUST LIKE A MATHS CLUB.
WAI...
woosh
BUT MIRUKA-TAN, WE ARE STILL NOT DONE HERE.
You're not going to do the piano duet?!
WE WILL CONTINUE TOMORROW.

...EH, THAT'S YOUTH.
CLOSE
USING THOSE PEANO AXIOMS, THE NATURAL NUMBERS CAN BE DEFINED.
THE NATURAL NUMBERS CAN BE DEFINED?!
..IS THERE ANY NEED TO DO SOMETHING LIKE THIS?

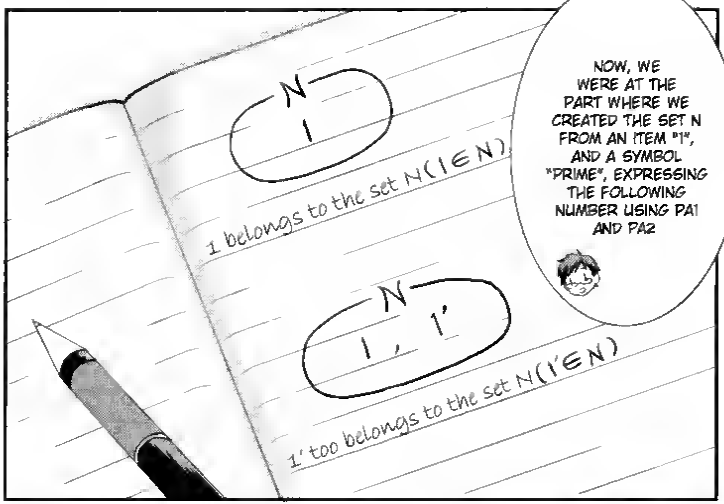
...IS WHAT I ALREADY KNOW!
DEFINING SUCH AN OBVIOUS THING... IS WHAT THIS IS ALL ABOUT?
TETRA-CHAN-
...TETRA-CHAN
NATURAL NUMBERS' DEFINITION AND ALL... 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
THAT'S RIGHT, BECAUSE PEANO, THE MATHEMA-TICIAN WHO THOUGHT OF THIS,
THOUGHT OF THE FUNDAMENTAL PROPERTIES OF NATURAL NUMBERS AND MADE THEM INTO THE PEANO AXIOMS
FUNDAMENTAL ...AXIOMS?
A-CHOO!
IT'S COLD HERE, LET'S GO TO GAKURA
Okay.
TO BE CONTINUED

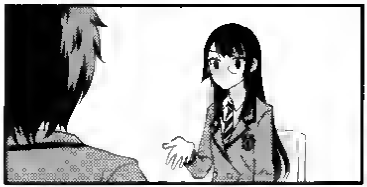

N
1, 1', 1'', 1''', 1'''' ...
1, 1', 1'', 1''', 1'''', ..
belong to the set N.
LOOK, USING THE TWO AXIOMS CALLED PA1 AND PA2 OUR NATURAL NUMBERS GROW BIGGER
TO PUT IT SIMPLY, YOU COULD SAY THAT THIS IS "A GAME OF PRETENDING NOT TO KNOW".
なるほど!!
INDEED!!
EVEN IF WE KNOW WHAT IT MEANS, WE WILL PRETEND THAT WE KNOW NOTHING OTHER THAN THE AXIOMS AND THE THINGS WE CAN DEDUCE FROM THEM..."
We use the symbol ' (prime) to express the number that follows and try to define a set called N. This set N will accordingly be {1, 2, 3, 4, 5...}! But we will pretend that we don't know this.
?
I don't know

"A GAME OF PRETENDING NOT TO KNOW" EH? SOUNDS NICE
ONLY THE THINGS WE KNOW FROM THE AXIOMS ACCUMULATE ONE BY ONE.
AND WE GO ON, USING LOGIC, TO BUILD UP EVEN THE THINGS THAT ARE USUALLY TAKEN AS GIVEN
UH-HUH, IN OTHER WORDS, IT COULD BE EXPLAINED LIKE THIS:
● You can use the axioms.
● You can also use things you logically conclude from the axioms.
● You can use the axioms repeatedly.
● Thus, we define the set N.
clatter
THAT'S RIGHT. WE ARE EXPECTING THE SET N, WHICH WE CREATE LOGICALLY USING THE AXIOMS, TO BECOME A SET OF ALL THE NATURAL NUMBERS.
AS EXPECTED, SENPAI'S EXPLANATION IS REALLY EASY TO UNDERSTAND!

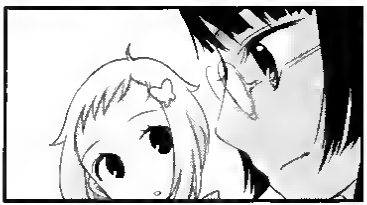
...THERE IS NO ONE OTHER THAN SENPAI FOR THIS!
Umm...
THERE IS A FAVOUR I WANT TO ASK FROM YOU, SENPAI!!
WELL...!
?
YO.

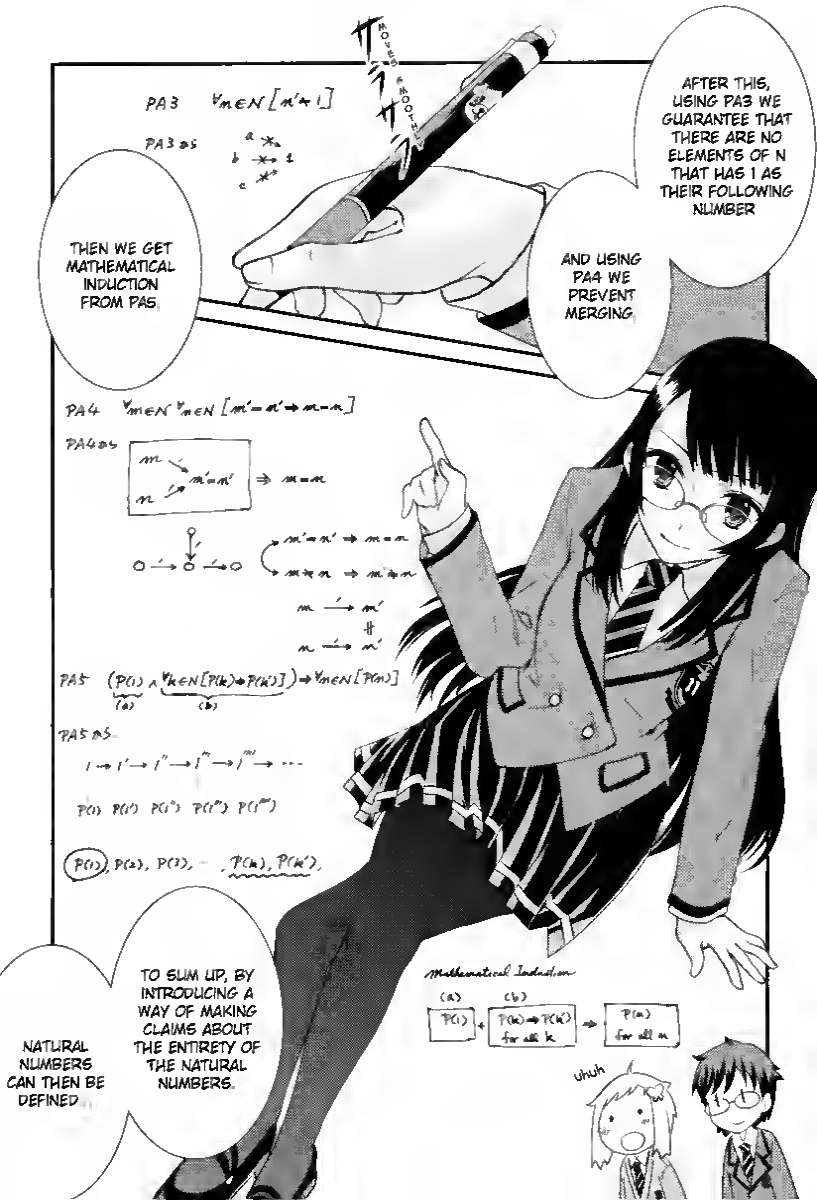
HERE YOU ARE.
MIRUKA-SAN
HMM.
PEANO'S AXIOMS, EH?


N
1
1 belongs to the set N(1∈N).
N
1 , 1'
1' too belongs to the set N(1'∈N)
NOW, WE WERE AT THE PART WHERE WE CREATED THE SET N FROM AN ITEM "1", AND A SYMBOL "PRIME", EXPRESSING THE FOLLOWING NUMBER USING PA1 AND PA2



Oh,
okay




PA3 $\forall_{n \in N}[n' \neq 1]$
PA3 as
THEN WE GET MATHEMATICAL INDUCTION FROM PA5.
AFTER THIS, USING PA3 WE GUARANTEE THAT THERE ARE NO ELEMENTS OF N THAT HAS 1 AS THEIR FOLLOWING NUMBER
AND USING PA4 WE PREVENT MERGING
PA4 $\forall_{m \in N} \forall_{n \in N}[m'=n' \Rightarrow m=n]$
PA4 as
$m' = n' \Rightarrow m = n$
$m \neq n \Rightarrow m' \neq n$
PA5 $(P(1) \wedge \underbrace{\forall_{k \in N}[P(k) \Rightarrow P(k')]}_{(b)}) \Rightarrow \forall_{n \in N}[P(n)]$
(a)
PA5 as
$1 \to 1' \to 1'' \to 1''' \to 1'''' \to \cdots$
$P(1)$ $P(1')$ $P(1'')$ $P(1''')$ $P(1'''')$
$P(1), P(2), P(3), \cdots, P(k), P(k'), \cdots$
Mathematical Induction
(a) $P(1)$
(b) $P(k) \Rightarrow P(k')$ for all k
$\Rightarrow$ $P(n)$ for all n
TO SUM UP, BY INTRODUCING A WAY OF MAKING CLAIMS ABOUT THE ENTIRETY OF THE NATURAL NUMBERS.
NATURAL NUMBERS CAN THEN BE DEFINED.
Uhuh

...USING ONLY 5 AXIOMS
YOU CAN SAY THAT THE PEANO AXIOMS ARE WHAT LOGICALLY DEFINES THE ENTIRE SET OF NATURAL NUMBERS WE THINK OF.
WE CAN REALLY DO SOMETHING LIKE THIS?
THAT'S THE POWER OF "SETS AND LOGIC".

HOWEVER, WHEN IT'S CONDITIONS OR LOGIC... I'M SOMEWHAT BAD AT THIS.
THAT'S AMAZING.
INCREDIBLY AMAZING.
IT MIGHT POSSIBLY BE THAT YOU ARE NOT GOOD AT "DEBATE".
I GET IT NOW, YOU ARE THE EXACT OPPOSITE OF YURI ON THIS POINT.
YURI SEEMS TO ENJOY THE FEELING OF LOGIC'S "DEBATES" WHICH ARE SETTLED BY TALKING POWERFULLY.
もくもく
SMOKE COMING OUT
What is it, Onii-chan? Are you talking about Yuri?

Onii-chan, you're with all the girls again!
annoyed
BUT I REALLY UNDERSTAND THE FEELING TETRA-CHAN IS TALKING ABOUT.
squeeze
YOU RESEMBLE ME A LITTLE AT BEING A WORRY-WART.
Wait, we're not done talking ye...
squeeeeze
Squeeeze
Y-Y-Y-Y-YOU REALLY THINK SO ..?
Wait, onii-chan? Onii-chaaan?

"SETS AND LOGIC" ...EH?
BY THE WAY TETRA, WERE YOU TRYING TO SAY SOMETHING?
NOW THAT YOU MENTION IT...
WHAT COULD IT HAVE BEEN, TETRA-CHAN?
UW-WAAH?!
U...
UM...
WELL!

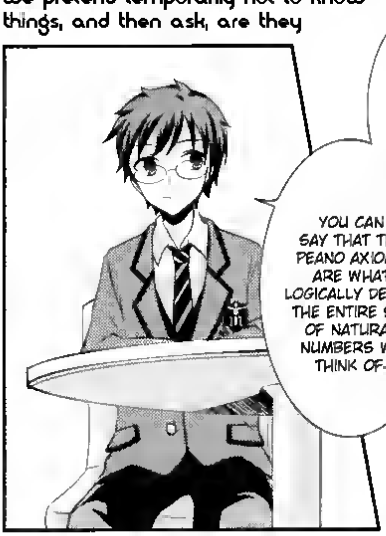
SENPAI!
WOULD YOU LIKE TO BECOME A TEACHER?!
A TEACHER?!

2nd period

The game of pretending not to know (Peano axioms)

Peano the mathematician thought of the Peano axioms in order to study the natural numbers (1, 2, 3...). An axiom means a statement held without proof, going by this, Peano proposed 5 axioms from PA1 to PA5.

We can usually do calculations with the natural numbers. Because we take them for granted, studying the natural numbers from basics is a difficult thing. Mathematics values logic very much. Getting through with "because it is natural" is impossible. The underlying assets of mathematics are "sets and logic" as Miruka-san also said. In modern maths, sets and logic are always used to establish the basis. In mathematics, we pretend temporarily not to know even the most natural things, and then ask, are they actually logically sound. This approach is very important in mathematics. This attitude is what really creates mathematics' strictness and charm.


USING ONLY 5 AXIOMS,
YOU CAN SAY THAT THE PEANO AXIOMS ARE WHAT LOGICALLY DEFINE THE ENTIRE SET OF NATURAL NUMBERS WE THINK OF.

Mathematical Girls
Godel s Incompleteness
Theorems

Mathematical Girls

c02

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Period 3: Dominoes of memories

UMM... SENPAI, WOULD YOU LIKE TO BECOME A TEACHER?!
A TEACHER ?!
AS A MATTER OF FACT, I RECENTLY BECAME GOOD FRIENDS WITH THE MIDDLE SCHOOL STUDENTS WHO WENT TO THE CULTURAL FESTIVAL.
THEY HAVE BEEN TELLING ME THAT THEY GOT INTERESTED IN MATHEMATICS AND WOULD LIKE TO HEAR MORE STORIES.
WHAT DO YOU MEAN, TETRA-CHAN?
WAAH!
S... SORRY.

FOR THAT REASON, I WANTED US TO DO A MATHS CLASSROOM AIMED AT MIDDLE SCHOOL STUDENTS.
I WAS WONDERING IF I COULD ASK SENPAI FOR THE FAVOUR OF TAKING THAT TEACHER ROLE...
CLENCH
SFX: BA-DUMP BA-DUMP
S... SOMETHING LIKE THIS IS IMPOSSIBLE FOR ME.
EH?

BUT-
BUT-
AT THAT TIME?
IT WAS YOU, SENPAI, WHO TAUGHT ME THE JOY OF MATHEMATICS AT THAT TIME.
TH... THAT'S RIGHT.
I'M SORRY.. FOR SAYING THIS SO SUDDENLY.

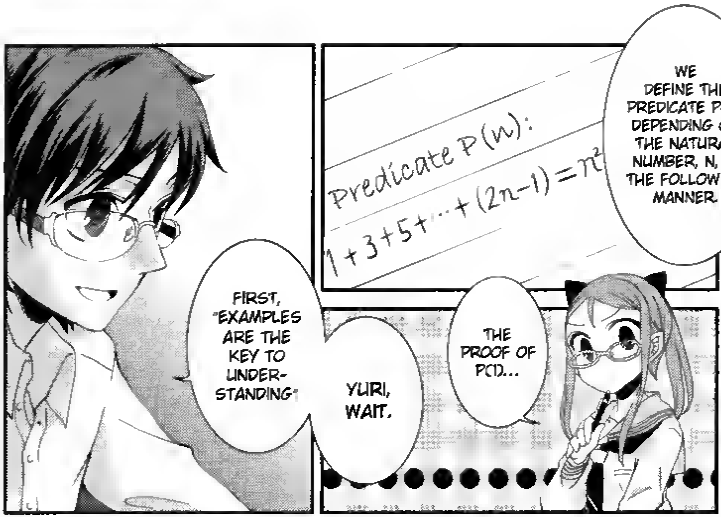
TETRA-CHAN...?
BOW
EXCUSE ME FOR TODAY.
THE REASON TETRA STARTED LOVING MATHEMATICS, WASN'T THAT YOU?
SOMEONE LIKE ME...

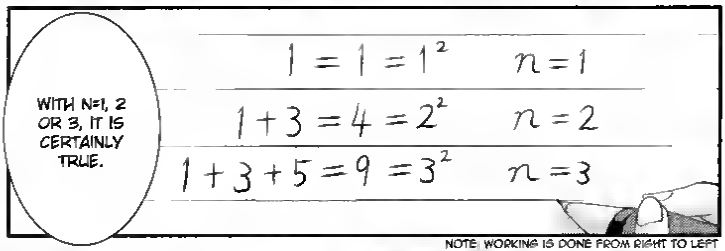
AH-
ENOUGH-
THAT'S ANNOYING.
REALLY ANNOYING.
YURI DOESN'T UNDERSTAND ANYTHING, IT'S ANNOYING~
SFX PISSED!!
WHAT'S WRONG WITH YOU, YURI?
LISTEN, ONII-CHAN. A BAD GUY, YOU SEE...
...BOASTFULLY TALKED TO YURI ABOUT SOMETHING CALLED MATIMATICAL-INDUKSHAN.
MATHEMATICAL INDUCTION, EH?

IN THAT CASE, THEN I SHOULD HAVE SOME MATERIAL I MADE SOME TIME AGO.
WHAT'S CALLED MATHEMATICAL INDUCTION IS A POWERFUL WEAPON.
FOR PROVING ARBITRARY PROPOSITIONS REGARDING THE ENTIRETY OF THE NATURAL NUMBERS
Found it
THIS IS WHAT I WAS TALKING ABOUT.
Grasping the Infinite~ A study of Mathematical Induction~
IT IS SOMETHING THAT GOES BACK MANY YEARS.
Grasping the infinite~ A study of Mathematical Induction~

IN MATHEMATICAL INDUCTION THERE ARE TWO STEPS.
IN STEP A, WE PROVE PROPOSITION P(1); WE CAN SAY THIS IS THE STARTING POINT.
AND IN STEP B, FOR ANY NATURAL NUMBER, K,
WE PROVE THAT "IF P(K) HOLDS THEN P(K+1) ALSO HOLDS"
IF WE ARE ABLE TO PROVE BOTH STEP A AND STEP B,
WE WILL BECOME ABLE TO PROVE THAT FOR ALL NATURAL NUMBERS, N, "P(N) HOLDS."
THAT'S A PROOF BY MEANS OF MATHEMATICAL INDUCTION.

Problem:
Prove the following is true for any natural number, n.
$1+3+5+\cdots+(2n-1)=n^2$
SQUEAK
LET'S TRY TAKING A LOOK USING A SPECIFIC PROBLEM.
SO ONII-CHAN HAS PERFORMED SOME MATHEMATICS!
THAT'S COOL, NYA
SINCE IT'S A SIMPLE EXAMPLE, LET'S TRY THINKING TOGETHER WHILE SOLVING IT.


Predicate P(n):
1+3+5+…+(2n-1)=n²
WE DEFINE THE PREDICATE P(N) DEPENDING ON THE NATURAL NUMBER, N, IN THE FOLLOWING MANNER.
FIRST, "EXAMPLES ARE THE KEY TO UNDER-STANDING"
YURI, WAIT.
THE PROOF OF P(1)...



NOTE: WORKING IS DONE FROM RIGHT TO LEFT

excited
AND NOW, WE PROVE STEP A AND STEP B IN ORDER.
FOR STEP A, WE FIRST PROVE P(1)
THE CLAIM, P(1) IS, IN OTHER WORDS, P(1) 1 = 1², THEREFORE IT IS CERTAINLY TRUE.
P(1): 1 = 1²
WITH THIS WE HAVE BEEN ABLE TO PROVE STEP A.
NEXT IS THE PROOF FOR STEP B.

THAT'S IT, YURI!
KO, T:—!
SFX WHOOPEE-T
UPON THAT ASSUMPTION, THIS IS TRUE!
Assuming true for P(k):
1+3+5+…+(2k−1)
=k²
SO DO WE ASSUME THAT FOR THIS K, P(K) IS FEASIBLE, NYA?
WE PROVE THAT FOR A NATURAL NUMBER, K, IF WE HAVE P(K), THEN P(K+1) ALSO HOLDS

$$1+3+5+\ +(2k-1)+(2(k+1)-1)=k^2+(2(k+1)-1)$$

Remove the brackets. $=k^2+2(k+1)-1$

Remove the brackets once more. $=k^2+2k+2-1$

Combine the constant parts. $=k^2+2k+1$

Factorise. $=(k+1)^2$

EXACTLY YURI, THAT'S ADMIRABLE!
THAT'S TO SAY, IT WAS POSSIBLE FOR OUR OBJECTIVE P(K+1) TO COME FROM THE PROPOSITION WE ASSUMED, NAMELY P(K).
WHOA! ONII-CHAN?
DOESN'T THIS MATCH THE P(K+1) FORM?
WITH THIS WE HAVE PROVED STEP B.
LIKE THIS, BOTH STEP A AND STEP B ARE DONE.

USING MATHEMATICAL INDUCTION, IT HAS BEEN POSSIBLE TO PROVE P(N) FOR ANY NATURAL NUMBER N.
TO SUM UP, THIS EQUATION HOLDS FOR ANY NATURAL NUMBER, N.
$=k^2+$
$=(k+1)^2$
$1+3+5+\cdots+(2n-1)=n^2$
SQUEAK
AND THAT'S EXACTLY WHAT I WANTED TO DEMONSTRATE.
— Q.E.D.
SQUEAK

*Q.E.D (Quod Erat Demonstrandum) A sign used at the end of a proof

$=k^2+2(k+1)-1$
$=k^2+2k+2-1$
$=k^2+2k+1$
$=(k+1)^2$
$3+5+\cdots+(2n-1$
— Q. E. D.

CLAP
CLAP
CLAP

AND THAT'S MATHEMATICAL INDUCTION
IT KIND OF RESEMBLES DOMINO TOPPLING, ONII-CHAN!
Ah!
THAT'S RIGHT, IT IS EXACTLY DOMINO TOPPLING.
HUH, IT FEELS LIKE I'VE HEARD THOSE WORDS SOMEWHERE BEFORE....
MATHEMATICAL INDUCTION CAN BE THOUGHT OF LIKE THIS, FIGURATIVELY.
WHAT YOU WANT TO PROVE CONSISTS OF "DOMINOES LINED UP IN A ROW WHICH COLLAPSE ENTIRELY".

Aye!
STEP (A) IS EQUIVALENT TO "THE FIRST DOMINO THAT FALLS".
Start
Yeah!
AND STEP B IS EQUIVALENT TO "IF THE K^TH DOMINO COLLAPSES, THE K+1^TH DOMINO GOES DOWN".
WHICH MEANS. "IF A DOMINO COLLAPSES, THE NEXT DOMINO DOES TOO".
k+1
k
OOH~!
INDEED, THAT'S RIGHT!

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33
IT IS A STATIC IMAGE.
MATHE-MATICAL INDUCTION IS USED TO SHOW US THE TRUTH OF PROPOSITIONS REGARDING EVERY SINGLE NATURAL NUMBER.
INSTEAD OF SCANNING THE NATURAL NUMBERS ONE-BY-ONE,
YOU GRASP THE ENTIRETY OF THE NATURAL NUMBERS FROM K=1 ALL AT ONCE.
CLENCH
USING THE POWER OF LOGIC YOU FIGHT AGAINST THE INFINITY CALLED THE NATURAL NUMBERS.

THE POWER OF MATHEMATICS AND LOGIC!
ONII-CHAN, YURI WAS MOVED.
I WAS MOVED.

Umm
EXCUSE ME.
It is break-time now.
MAY I HAVE A MOMENT OF YOUR TIME?
THERE'S A PART I DON'T UNDERSTAND AND...
...AND THAT'S HOW IT IS
I SEE NOW!
THANK YOU VERY MUCH

MATHEMATICAL INDUCTION IS ALMOST LIKE TOPPLING DOMINOES
DOMINOES, YOU SAY .?
THAT'S AN EXCELLENT METAPHOR, YOU HAVE GOOD TASTE.
HOWEVER, I WANT TO WORK IN A JOB RELATED TO COMPUTERS IN THE FUTURE,
SO I THOUGHT THAT MATHS WOULD BE NECESSARY AFTER ALL.
WAAH-
NO, TO BE HONEST I'M NOT REALLY GOOD AT MATHS..
BLUSH

MORE...
.. AND MORE!
I WANT TO LOVE MATHEMATICS!
WHEN YOU TALKED 'BOUT USING THE POWER OF MATHEMATICS AND LOGIC TO FIGHT AGAINST THE INFINITY CALLED THE NATURAL NUMBERS, IN YOUR SPEECH EARLIER, I WAS MOVED.
EVEN THOUGH THAT'S THE WAY I AM.

NOW I REMEMBER...
THE GIRL FROM THAT TIME... WAS THAT TETRA-CHAN?!
MUMBLE
I WANT TO LOVE MATHE-MATICS...
AS EXPECTED, ONII-CHAN IS REALLY GOOD AT TEACHING, NYA
nyahaha!
I WISH YOU WERE A TEACHER
A TEACHER...

I SHALL TAKE IT UP, TETRA-CHAN.
IS THAT SO?
I SEE NOW...
CLENCH

Staff Room
I WONDER WHERE TETRA-CHAN IS...
Proficiency test, high ranking students
OH, IT'S ALREADY TIME FOR THE RESULTS FROM THE PROFICIENCY TEST.
CHATTER
CHATTER

H...HEY.
GOOD MORNING, TSUNOMIYA
Proficiency test high ranking students
AH
WHAT'S WRONG?
IT NOT LIKE Y..?!

3rd period

Mathematical Induction

Mathematical induction is one of the methods used to prove propositions regarding the naturalnumbers.

Since the natural numbers (1, 2, 3, ...) are infinite, if you prove something for n=1, then prove it for n=2, and then prove it for n=3... no matter how many times you do this it won't serve any purpose. Because no matter how many times you prove it, an end is never reached

Mathematical induction proves propositions regarding the infinite natural numbers using no more than two steps. In the 3rd period we explained two steps called step A and step B.
Step A is the starting point. It proves P(1) which means in the case of n=1.

Step B proves that, for any natural number, k, "If P(k) is true, then P(k+1) holds true as well".

With those two steps, just like domino toppling, the truth of P(n) is also proved for any natural number, n. Fighting against infinity with just two steps, don't you think mathematics is marvellous?

Mathematical Girls
Godel s Incompleteness
Theorems

Mathematical Girls C03

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Period 4: **A Set For Two**

...THERE-FORE...
...THERE IS AN IMPORTANT CONCEPT FOR DEALING WITH INFINITY. DO YOU KNOW ABOUT IT?
WHO KNOWS ...?
WHAT'S WRONG? THIS ISN'T LIKE YOU.
WASN'T INTERPRETING MATHEMATICS YOUR DOMAIN?

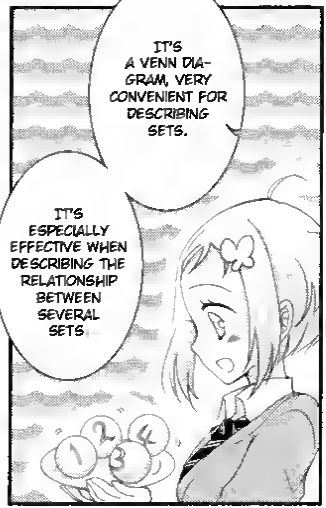
I DON'T KNOW WHAT I DON'T KNOW
IT SEEMS IMPOSSIBLE TODAY.
SORRY, I'LL BE GOING HOME FIRST.
SLAM

YEAH, IT LOOKS LIKE HE'S FEELING DOWN BECAUSE HE FELL OFF THE RANKINGS FOR THE FIRST TIME FOR THAT PROFICIENCY TEST A FEW DAYS AGO.
SENPAI'S BEEN ACTING STRANGE FOR THE LAST FEW DAYS, HASN'T HE?
Perhaps it's because I made an unreasonable request...
SENPAI, WHO'S A REGULAR ON THE RANKING LIST?
EH?
DIDN'T YOU GET A MENTION IN TWO SUBJECTS, ENGLISH AND MATHEMATICS, TETRA?
Oh.
YES IT WAS EMBARRASSING.

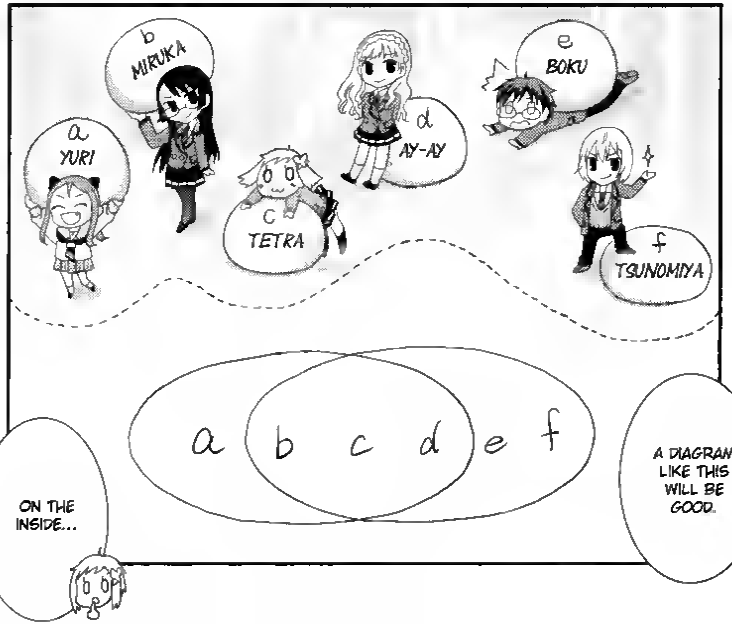
IN THE END, WHAT'S GOING TO HAPPEN WITH THE LECTURE?
IT MUST HAVE BEEN REALLY HARD FOR SENPAI.
I'M WORRIED.
BUT REALLY... THIS WAS THE THING TROUBLING HIM.
LET'S CONTINUE THEN... WE WERE TALKING ABOUT INFINITY.
IS THAT SO?
ABOUT THE MATHS CLASSROOM, I WAS THINKING. IT'S A BIT ARROGANT, BUT I'LL TRY TO FILL THAT POSITION MYSELF.

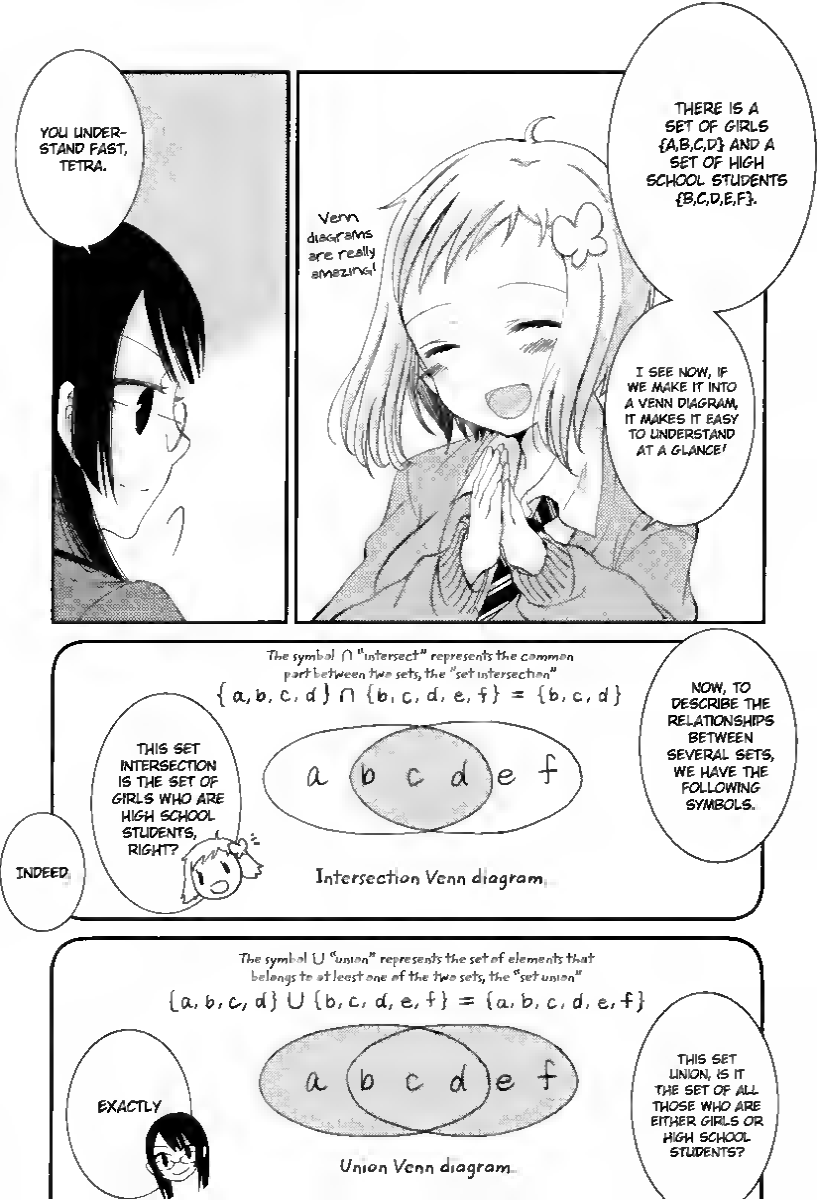
YES!
WITHOUT ENUMERATING THE NUMBERS ONE-BY-ONE, BY USING THE PEANO AXIOMS OR MATHEMATICAL INDUCTION,
GRASPING THE NATURAL NUMBERS ALL AT ONCE IS EXCITING!
HMM. . LET'S TALK ABOUT 'SETS' AND 'LOGIC' THEN
SETS, YOU SAY?
THAT'S RIGHT. FOR EXAMPLE, THE COLLECTION OF ALL THE INTEGERS WHICH ARE GREATER THAN 1 AND ARE MULTIPLES OF 2 WILL BE CALLED "THE SET OF ALL THE MULTIPLES OF 2".
NOW, WE WRITE THEM LIKE THIS
{2, 4, 6, 8, 10, 12, 14, 16, ...}
THIS IS A SET.
HERE ARE THE RULES FOR DESCRIBING SETS.
Rule
• Specific examples of the elements are separated by commas (,) and put side by side
• Elements can be put in any order
• If the elements are infinite, dots (...) are added
• Enclose everything within curly brackets { { } }.

WAAH!
ELEMENTS OF A SET ARE NOT NECESSARILY NUMBERS.
FOR EXAMPLE, A SET ITSELF COULD BE AN ELEMENT OF ANOTHER SET.
TO SHOW THAT SOMETHING IS AN ELEMENT OF A SET, WE USE THE SYMBOL ∈.
LOOK AT THIS, FOR EXAMPLE
10 ∈ {2, 4, 6, 8, 10, 12, 14, 16, ... }
IT DID! LOOK, HERE IT IS!
PA1 1 ∈ N
Oh!
THIS SYMBOL, DIDN'T IT APPEAR THAT TIME IN THE PEANO AXIOMS?
flip
flip


IT'S A VENN DIAGRAM, VERY CONVENIENT FOR DESCRIBING SETS.
IT'S ESPECIALLY EFFECTIVE WHEN DESCRIBING THE RELATIONSHIP BETWEEN SEVERAL SETS.
1
2
3
4

YES, THAT'S EXACTLY IT.
WHAT THIS EXPRESSES IS THAT "1 IS AN ELEMENT OF N".
1
N
IF WE REPRESENT THIS USING A DIAGRAM, IT CAN BE DRAWN LIKE THIS
N
1
1 belongs to the set N (1 ∈ N)

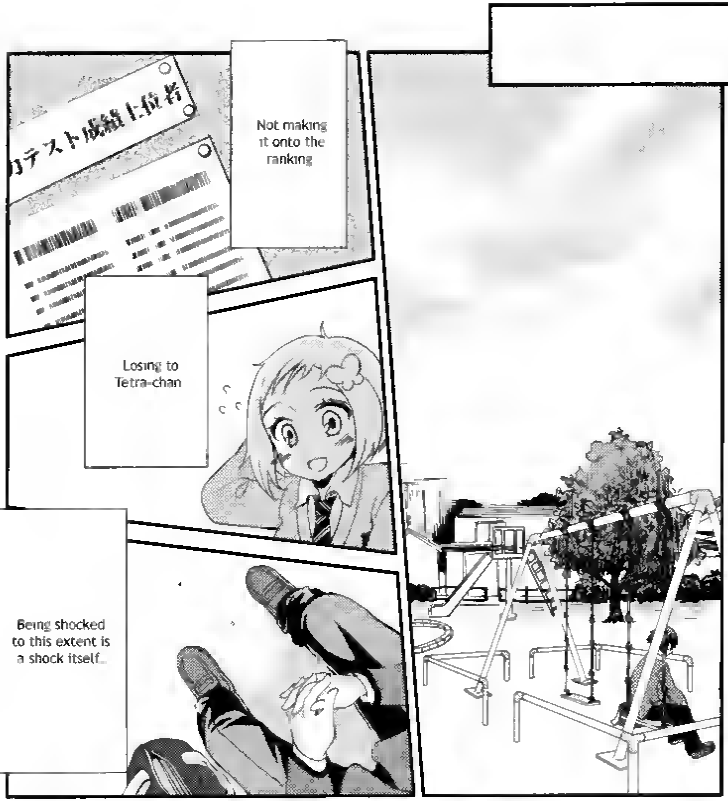

a
YURI
b
MIRUKA
c
TETRA
d
AY-AY
e
BOKU
f
TSUNOMIYA
a b c d e f
ON THE INSIDE...
A DIAGRAM LIKE THIS WILL BE GOOD.


YOU UNDERSTAND FAST, TETRA.
Venn diagrams are really amazing!
THERE IS A SET OF GIRLS {A,B,C,D} AND A SET OF HIGH SCHOOL STUDENTS {B,C,D,E,F}.
I SEE NOW, IF WE MAKE IT INTO A VENN DIAGRAM, IT MAKES IT EASY TO UNDERSTAND AT A GLANCE!
The symbol ∩ "intersect" represents the common part between two sets, the "set intersection"
{a, b, c, d} ∩ {b, c, d, e, f} = {b, c, d}
a b c d e f
Intersection Venn diagram
NOW, TO DESCRIBE THE RELATIONSHIPS BETWEEN SEVERAL SETS, WE HAVE THE FOLLOWING SYMBOLS.
THIS SET INTERSECTION IS THE SET OF GIRLS WHO ARE HIGH SCHOOL STUDENTS, RIGHT?
INDEED.
The symbol ∪ "union" represents the set of elements that belongs to at least one of the two sets, the "set union"
{a, b, c, d} ∪ {b, c, d, e, f} = {a, b, c, d, e, f}
a b c d e f
Union Venn diagram
EXACTLY
THIS SET UNION, IS IT THE SET OF ALL THOSE WHO ARE EITHER GIRLS OR HIGH SCHOOL STUDENTS?

OH!
I'M REALLY EXCITED!
NOW, USING THIS, LET'S GRASP THE INFINITE ALL AT ONCE
WITH THIS, WE HAVE PINNED DOWN THE FUNDAMENTAL RULES OF DESCRIBING SETS
{1, 2, 3, 4, 5}
FIRST, WE CAN LIST THE ELEMENTS LIKE THIS, WE CALL DEFINING THE SET AN "EXTENSIONAL DEFINITION"
YOU'RE RIGHT!
Extensional definition
{2, 4, 6, 8, 10, 12, 14, 16, ...}
{n | is 2multiple of2}
Intensional definition
FOR EXAMPLE, THE DEFINITION FOR "THE COLLECTION OF ALL THE MULTIPLES OF 2" EARLIER IS JUST LIKE THAT.
THE CONDITIONS THAT HAVE TO BE MET BY ELEMENTS . ?
ON THE OTHER HAND, WE CAN ALSO DEFINE SETS BY WRITING THE CONDITIONS THAT HAVE TO BE MET BY ELEMENTS AS PROPOSITIONS
WE CALL THIS AN "INTENSIONAL DEFINITION".

THE MULTIPLES OF 2 ARE INFINITE
{n | n is a multiple of
BUT, BY WRITING LIKE THIS, IT WAS POSSIBLE TO DROP THOSE INTO A SET.
A...AND THEN?
gulp
THAT'S TO SAY, IT'S POSSIBLE FOR THEM TO BECOME THE TARGET OF MATHEMATICAL STUDY, SUCH AS SET OPERATIONS
THAT MEANS A SET COULD REPRESENT ANYTHING, WHATEVER IT WAS?
Eeh?
THAT'S RIGHT.
EVEN THOSE CONCEPTS THAT SEEM ABSTRACT. IF THEY CAN BE DESCRIBED BY A LOGICAL EXPRESSION THEY CAN BE SUCCESSFULLY FITTED INTO THE FORM CALLED "A SET" AND THEN BECOME THE TARGET OF MATHEMATICAL STUDY.
BUT BEING CAREFUL NOT TO RAISE CONTRADICTIONS IS NECESSARY, WHETHER IN ALGEBRA, GEOMETRY OR ANALYSIS.
THEREFORE...
...MATHEMATICS ITSELF CAN BECOME A TARGET OF MATHEMATICAL STUDY.

BECAUSE, IF WE USE SETS AND LOGIC, IT IS EVEN POSSIBLE TO "MATHEMATICISE MATHEMATICS"
MATHEMATICISE MATHEMATICS? THAT'S AMAZING!
WE CAN ALSO SAY THAT THIS IS THE STARTING POINT OF GÖDEL'S INCOMPLETENESS THEOREM
THAT'S RIGHT, WITH THIS WE HAVE ONE WEAPON FOR CHALLENGING GÖDEL'S INCOMPLETENESS THEOREM, SETS AND LOGIC, IN OUR HANDS.
Weapon!!
gleam
SETS AND LOGIC
AMAZING ...THAT'S REALLY AMAZING!
I WAS REALLY MOVED!


Not making it onto the ranking
Losing to Tetra-chan
Being shocked to this extent is a shock itself...

Did I become so fragile that I would break down because of something like this?

Pathetic feeling
Pathetic self
Even though I know that maths by itself is not everything...
But...
But for me...

TETRA-CHAN.

YOU DIDN'T GO HOME EARLY, SENPAI?
What are you doing in a place like this?
I WAS COOLING MY HEAD FOR A BIT. WHAT ABOUT YOU, TETRA-CHAN, WHY ARE YOU HERE?
I'M DOING SOME VOCAL EXERCISES PREPARING FOR TOMORROW.
I'M NOT GOOD IN FRONT OF AN AUDIENCE
THAT'S WHY I'M REHEARSING HERE
AH, THIS IS ABOUT THE MATHS CLASS-ROOM.

THERE IS THAT TOO, BUT IT'S ALL RIGHT.
...THE TEST DIDN'T GO WELL, RIGHT...?
THAT'S A LIE.
SENPAI IS SUFFERING.
I'M JUST FED UP WITH MYSELF.

I LEARNT THE JOY OF MATHEMATICS FROM SENPAI.
IT'S NOT JUST ME MAY LOTS OF PEOPLE, THROUGH SENPAI,
BE ABLE TO KNOW THE JOY OF LEARNING MATHEMATICS.

I WISH THIS FROM THE BOTTOM OF MY HEART.
SWISH

See ya later
PLEASE COME AND SEE THE MATHEMATICS LECTURE TOMORROW, SENPAI
?

20
21
Mathematics Lecture
27
I WAS THINKING THAT MIRUKA-SAMA WAS GOING TO TAKE THE TEACHER ROLE
NO, I'M NOT SUITED TO BEING A TEACHER
THAT IS HIS ROLE
IS THAT SO...?
THAT ONII-CHAN, THOUGH. I WONDER IF HE'S NOT COMING AFTER ALL.
HMM...

TREMBLE
TETRA-SAN IS REALLY STIFF...
MIRUKA-SAMA, TETRA-SAN HAS APPEARED!
SQUEEZE
E... Everyone...
Th..
Th..
Thank you very much for gathering today.

APPLAUSE
BLUSH
To... to... to... today's mathematical classroom...
Wi... wi... wi... will be about infinity!
HMM.
I wonder if Tetra-san is okay
SINCE I WILL EXPLAIN USING THE PROJECTOR,
PLEASE LOOK AT THE SCREEN IN FRONT OF YOU
POOF

0.999... = 1
LIGHT
TODAY'S LECTURE WILL BE ABOUT THIS EQUATION ON THE SCREEN.
NYANYA! WHAT IS THIS EQUATION!?
0.9 IS 1?!
clench

CHATTER
CHATTER
W. WHAT? IT BECAME PITCH BLACK.
DARK
UH... UHHH, WHAT?
click
click
THAT'S STRANGE WHAT SHOULD I DO ..!?
IT'S NO GOOD IT'S NOT LIGHTING UP
WHAT DO I DO? WHAT CAN I DO...?

BUZZ
BUZZ
SENPAI.
thud

4th period

Sets and Logic

In the 4th period, the topic was rules for expressing sets.
Miruka san explained to Tetra san what the symbols used in mathematics like {} or $\in$ or $\cup$ or $\cap$ represent.

Not being used to the symbols makes it very hard to read. When we feel that they are difficult, we unconsciously want to skip them, but taken one-by-one there is no way they will be difficult. While comparing the Venn diagram with your eyes, think well about the meaning those symbols reveal.

Did you notice that a Venn diagram also appeared in the beginning of 4th period (page 103)? In the set of people wearing glasses (GLASSES) were Miruka san and Yuri, and in the set of high school students (HIGHSCHOOL) we had Miruka san and Tetra chan. Thus Miruka san belonged to the common part of the two sets.

Like Miruka san said, in madern mathematics, even the abstract concepts can become the subject of research with "sets and logic". Because sets and logic are an important weapon for fighting mathematical problems.

When studying mathematics, let's think hard about the meanings the symbols express.

Mathematical Girls
Gödel's Incompleteness
Theorems

Mathematical Girls

C04

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Illustrator えらんと

Period 5: **My Destination**

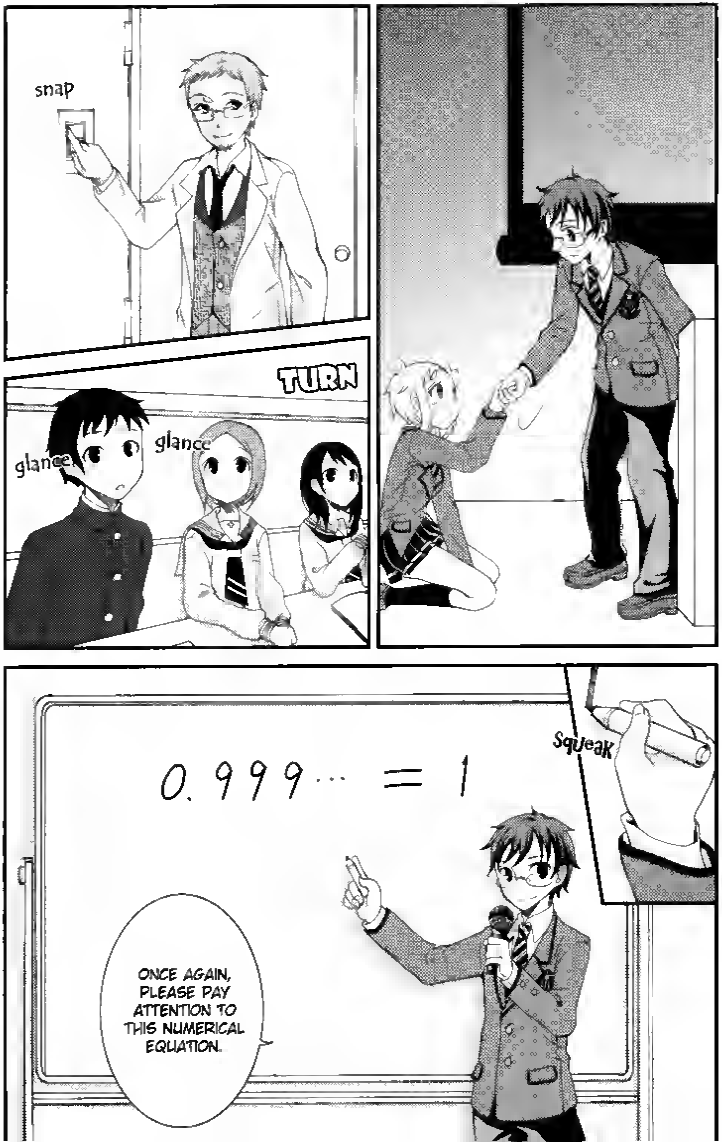
The atmosphere feels odd... and it's pitch-black
I wonder if it has already begun...?
The screan isn't lit? Could it be a mal-function?

Chatter
Chatter
BUT EVERYONE IS LOOKING... WHAT SHOULD I DO..?
IT'S IMPOSSIBLE FOR ME TO AD-LIB WITHOUT ANY MATERIALS..
Silent...
...HELP ME, SENPAI.
I learnt the joy of mathematics from senpai.

Not just me, though. Many people, through senpai,
are able to know the joy of learning mathematics.
STEP
STEP

Behold!
EVERYONE, I'M SORRY FOR BEING LATE.
I'M TODAY'S ASSISTANT LECTURER.

Hmm
Onii-chan?!


snap
TURN
glance
glance
0.999… = 1
Squeak
ONCE AGAIN, PLEASE PAY ATTENTION TO THIS NUMERICAL EQUATION.

LET ME SEE, UMM...
HUH?
TETRA-SENSEI, IS IT TRUE THAT THIS EQUATION IS CORRECT?
BUT IT'S 0.999... RIGHT?
Y...YES, IT IS TRUE.
ISN'T THAT A LITTLE BIT LESS THAN 1?

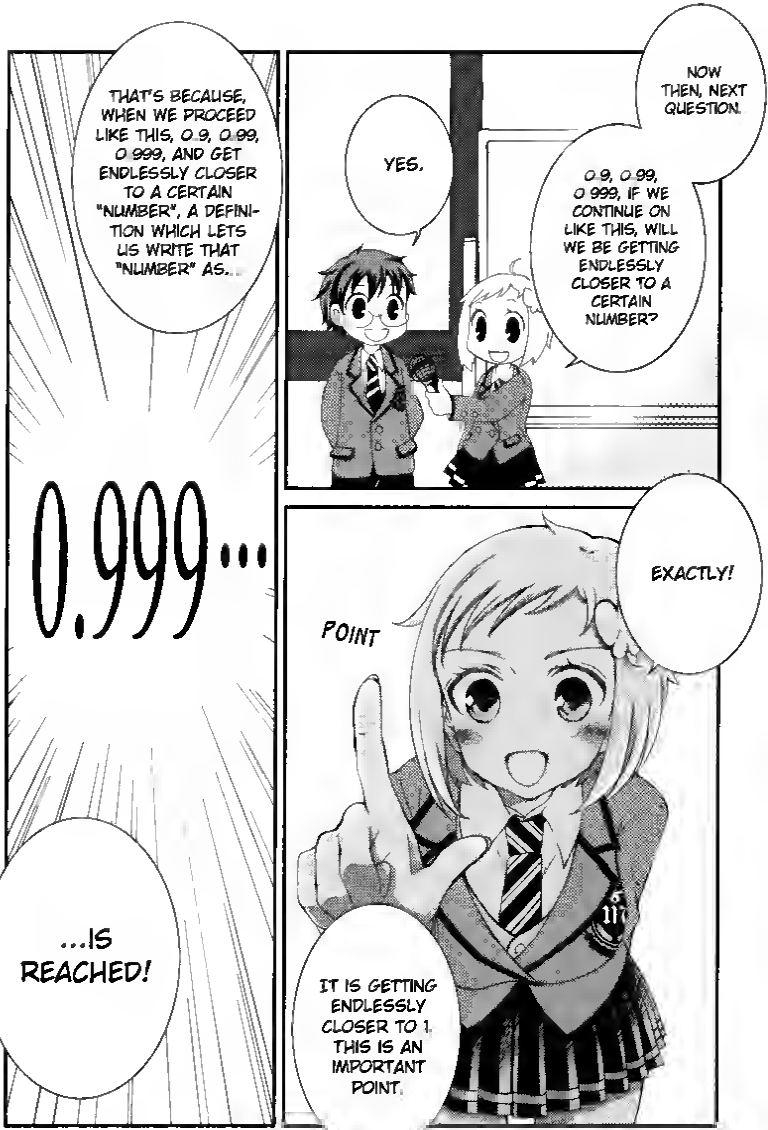
That's the spirit.
I... I THINK SO TOO
touched
WHAT DOES EVERYONE ELSE IN THIS THEATRE THINK?

YES, THAT'S RIGHT.
MIRUKA-SAMA, IT LOOKS LIKE TETRA-SAN IS ALRIGHT NOW.
SIGH~
SLIDE
PERHAPS, IF WE KEEP GOING OVER AND OVER UNTIL INFINITY, AT THE END THE DIFFERENCE WILL BECOME 0.000..., THAT'S TO SAY IT WILL BE 0 AND THIS WILL BE EQUAL TO 1?
MAYBE IF WE CONTINUE EXAMINING THIS "0.9, 0.99, 0.999" SEQUENCE, THE DIFFERENCE FROM 1 KEEPS GETTING SMALLER.
BUT, IF 0.999... AND 1 ARE WHAT WE ARE TALKING ABOUT, WHEN WE COMPARE THEM, THE CORRECT THING IS TO WRITE THAT 0.999 < 1, DON'T YOU THINK?
YES, BUT IF WE CONTINUE DOING "0.9 < 1", "0.99 < 1" OVER AND OVER UNTIL INFINITY, WON'T THIS MAKE "0.999... < 1"?
Hm
Mm

0.9, 0.99, 0.999, ...
• NO MATTER HOW FAR WE GO, IT IS STILL LESS THAN 1.
• WE GET AS CLOSE TO 1 AS WE LIKE.
IF WE ORGANIZE THE ISSUE EVERYONE IS DEBATING, WON'T IT LOOK LIKE THIS?
TETRA-SENSEI, WHICH OF THOSE TWO IS THE CORRECT ONE?
THAT'S A GOOD QUESTION! ACTUALLY, THEY ARE BOTH CORRECT.
Tetra-sensei

0.9
0.99
0.999
1
Squeak
IS THAT SO! I WILL TRY TO DRAW A DIAGRAM.
Vroom
0.999...
0.9
0.99
0.999
IF WE TRY WRITING IT LIKE THIS, THE SEQUENCE 0.9, 0.99, 0.999 WILL...
...GO ON WHILE ENDLESSLY GETTING CLOSER TO 1.
Assistant
Destination
IN OTHER WORDS, THE DESTINATION, IS EQUIVALENT TO 0.999...
THAT'S RIGHT. THE PLACE WHICH WE ARE TENDING TO,

THEN LET'S CHANGE OUR PERSPECTIVE FOR A WHILE AND TRY TO THINK USING A DIFFERENT APPROACH, SHALL WE?
SENPAI, PLEASE ANSWER THESE QUESTIONS WITH "YES" OR "NO".
BUT STILL, EVEN AFTER SEEING THAT DIAGRAM, I DON'T THINK 0.999... WILL BECOME 1.
0.9, 0.99, 0.999, IF WE CONTINUE ON LIKE THIS, WILL IT EVENTUALLY BECOME EQUAL TO 1?
NO.
LIKE SO! THAT'S THE SPIRIT. NOW LET'S GO.
YES.
THAT'S RIGHT, NO MATTER HOW MANY 9S WE PLACE AFTER 0.9, IT WILL STILL BE LESS THAN 1.
0.99999999999

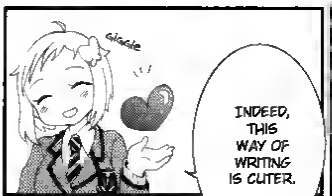

NOW THEN, NEXT QUESTION.
YES.
0.9, 0.99, 0.999, IF WE CONTINUE ON LIKE THIS, WILL WE BE GETTING ENDLESSLY CLOSER TO A CERTAIN NUMBER?
EXACTLY!
POINT
IT IS GETTING ENDLESSLY CLOSER TO 1. THIS IS AN IMPORTANT POINT.
THAT'S BECAUSE, WHEN WE PROCEED LIKE THIS, 0.9, 0.99, 0.999, AND GET ENDLESSLY CLOSER TO A CERTAIN "NUMBER", A DEFINITION WHICH LETS US WRITE THAT "NUMBER" AS...
0.999...
...IS REACHED!

AH!
IS THAT SO? IT'S CLEAR NOW!!
I WAS THINKING THAT MAYBE THE 0.999... DOESN'T COME OUT IN THE SEQUENCE.
YOU OVER THERE, WOULD YOU PLEASE EXPLAIN TO EVERYONE THE THING YOU NOTICED JUST NOW?
EH? EH? ERR... ME?

- $0.999\ldots$ REFRESENTS OUR "NUMBER"
- IF WE PROCEED WITH 0.9, 0.99, 0.999, WE GET ENDLESSLY CLOSER TO THAT "NUMBER"
- BUT EVEN IF WE GO ON WITH 0.9, 0.99, 0.999, THAT "NUMBER" WON'T COME OUT
- THE "NUMBER" REFRESENTED BY $0.999\ldots$ IS EQUIVALENT TO 1

Squeak

IF $0.999\ldots$ IS DIFFERENT FROM NUMBERS LIKE 0.999 OR 0.99 AND DOESN'T COME OUT IN THE SEQUENCE,
IT WOULD HAVE BEEN NICE IF IT WAS REWRITTEN AS ♡



THE POINT IS THE NOTATION.
THAT'S TO SAY..
0.999··· (IS EQUIVALENT TO 1)
0.999···9 (IS LESS THAN 1)
step
step
WE CAN SAY THAT THOSE TWO, THOUGH VERY SIMILAR, ARE ACTUALLY TWO DIFFERENT THINGS.
THUS THE EQUATION WHICH SHOWS THAT 0.999.. AND 1 EXPRESS EQUAL NUMBERS IS...

0.999・・・＝1
CORRECT!!
ooo～H
clap
clap
clap
clap

LET'S TRY WRITING THIS USING MATHEMATICAL EXPRESSIONS.
$\lim_{n\to\infty} a_n$
$\lim_{n\to\infty} a_n = A$
$a_n \to A$ as $n \to \infty$
We can write it like th-

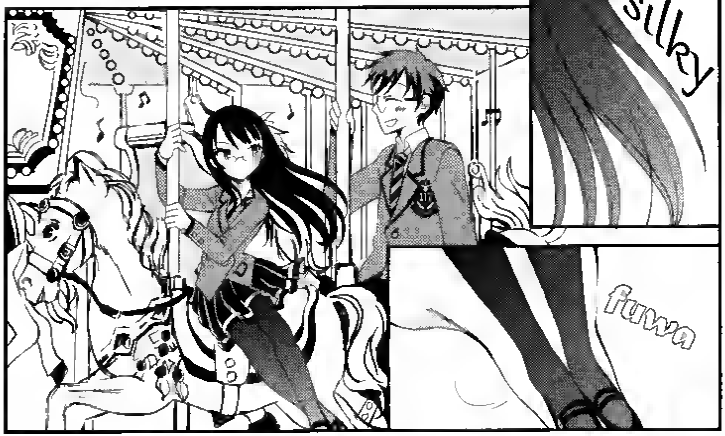
YURI-CHAN! THANK YOU!!
Bam!
OPEN
TETRA-SAN, GOOD JOB!
YEAH, YOU WERE SPLENDID.
YOU WERE A REALLY GOOD LEC-TURER, TETRA.

SE... SENPAI! MIRUKA-SAN!!
I... I, UUU...
UWAAAH

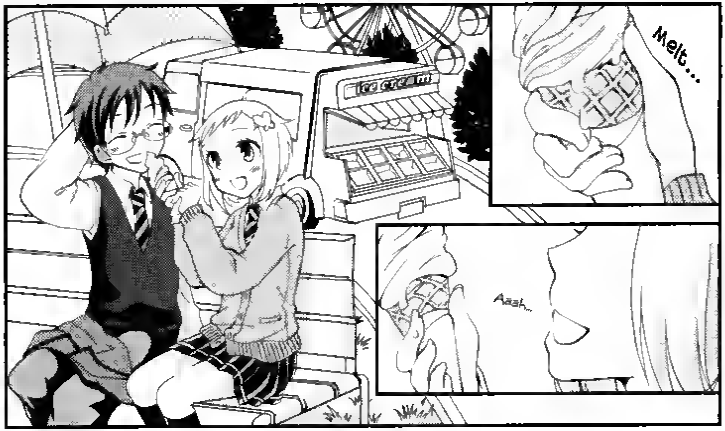
BECAUSE ONII-CHAN'S ENTRANCE WAS A LITTLE LATE, I WAS WORRIED ABOUT WHAT WAS GOING TO HAPPEN.
So...somehow it's already over
sob
sob
UH... SORRY
IT LOOKS LIKE YOU GOT BACK IN SHAPE, I WAS WORRIED.
THANKS TO TETRA-CHAN, I WAS ABLE TO REMEMBER THE IMPORTANT THINGS.

SE... SENPAI!
The joy of learning mathematics, the joy of teaching mathematics ...huh
COUGH.

WELL THEN, SHOULD WE HAVE A PARTY TO CELEBRATE?
I HAVE AMUSEMENT PARK TICKETS.
ふぅ!!
ga-
-pe
?
shocked
MIRUKA-SAMA... AMUSEMENT PARK...?


silky
funwa

shake
shake


Melt...
Aaah...

shake
shake shake
shake

Hmph
GLANCE
Dizzy
Continued in volume 2

5th period

Limits

Although "0.999... = 1" looks weird, it is a correct equation from a mathematical point of view.

In 4th period's column (page 129), we said "let's think hard about the meanings the symbols express". Here as well, it is necessary to think well about the meaning of the expression "0 999...".

Thinking of "0.999..." as "something where digits continue until infinity" is not correct. If we think like this, we will end up misunderstanding "0.999..." as if it were "something that changes". If we deal with infinity with our intuition, without using mathematics, we will end up in chaos.

"0.999..." expresses a "single number" with a fixed value. The sequence 0.9, 0.99, 0.999 describes the destination which we are endlessly getting closer to. The 0.9, 0.99, 0.999 sequence gets ever closer to "1". Therefore, "0.999... = 1" is a correct equation from a mathematical point of view. Like this, the destination point is mathematically called a "limit point". And the process of pursuing a limit is called "taking the limit".

In order to handle limits mathematically, it is necessary to study ε-δ logic (epsilon-delta logic). By using limits we become able to deal correctly with one of the aspects of infinity.

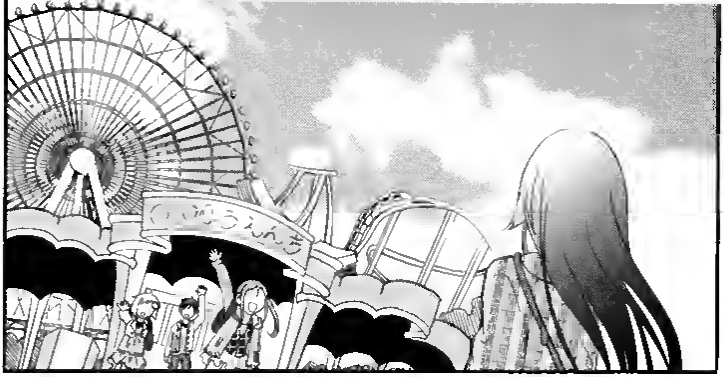
Not until the 19th century was mankind able to deal correctly with limits. That's because limits are that much of a difficult thing.

You too, when studying mathematics, why don't you try to challenge the world of infinity?

VOLUME 2 ADVANCE NOTICE BONUS MANGA.
Me
THE AMUSEMENT PARK TOGETHER WITH SENPAI...
Blush
I WONDER WHAT I SHOULD WEAR TO THE AMUSEMENT PARK TOMORROW
PILED
This one is cheerfully lively like the typical me
Oww
What should I do~?
?
SERIOUS?
I guess
A slightly mischievous active girl?
I guess the girlishly sexy look?

FWAP
Who?
It's cold
Onii-chaaan! Let's go to the amusement park!!
Wake up!
The appointed day
I was joking. You really believe like a doll
Ahh
Hmph
STRAIGHTEN
Onii-chan... how about this?
I got it
slip
You are treating me like a kid again~!!
Enough!
Onii-chan tie it!!
rattle
rattle

RATTLE
CLUNK
close
CLOSE!
SLIDE-



To Be Continued ... !!!

[Mathematical Girls Gödel's Incompleteness Theorems] Original author's postscript

Hello, this is Yuki Hiroshi, the original author.
This work is volume three of the Mathematical Girls series [Mathematical Girls Gödel's Incompleteness Theorems] which was kindly reorganized into a comical version by Matsuzaki Miyuki sensei.
Please enjoy the lively figures of Mathematical Girls' characters which couldn't often be seen in the original work.
It would really make me happy if everyone realizes the joy of learning mathematics through this work.
Enjoy!
Mathematical Girls series homepage ... http://www.hyuki.com/girl/

Postscript

Special thanks!!

Sorry for the messy handwriting.

Original author-sama Yuki Hiroshi sensei, thank you very much for your whole-hearted support!!
It is an honor to be able to participate in such a magnificent work.
Ever since I had the privilege of reading the original work I have now been able to savor to my heart's content the charm of mathematics which I was not able to know before.
The person in charge Nakamura-san because of your encouragement and guidance all the time, it was possible to reach this far.
The one who sucked up my extensive lack of maturity starting as my foodssistant and even helping with the literary creation work Reika-san, you were very reassuring! You saved me many times, thank you.
The friends who gave me advice, the one who made both my heart and body feel better in the osteopathic clinic Fujita-sensei.
And and.
You who have kept reading until now really, **thank you very much!!!** Volume 1 was Tetra-chan's turn but volume 2 will be Miruka-san's turn?! I would be happy to get the chance to meet you again!!2.3

Mathematics is amazing!

I'm really indebted to you.

What about Yumi?

2013.3 Matsuzaki Miyuki

Mathematical Girls
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